

“MR. CALCULUS” ANSWERS TO THE 2010 FORM B FREE RESPONSE QUESTIONS

BC 2

$$\frac{dx}{dt} = x'(t) = 14 \cos(t^2) \sin(e^t) \quad \text{and} \quad \frac{dy}{dt} = y'(t) = 1 + 2 \sin(t^2) \quad \text{for } 0 \leq t \leq 1.5$$

(a) The tangent line is vertical when $\frac{dx}{dt} = 0$ and $\frac{dy}{dt} \neq 0$. This occurs at $t = 1.145$ and $t = 1.253$.

(b) Let position of the particle be $(x(t), y(t))$. Then

$$x(1) = x(0) + \int_0^1 x'(t) dt = -2 + 11.315 = 9.315 \quad \text{and}$$

$$y(1) = y(0) + \int_0^1 y'(t) dt = 3 + 1.621 = 4.621.$$

$$\text{At } t = 1, \quad \left. \frac{dy}{dx} \right|_{t=1} = \frac{dy/dt}{dx/dt} \Big|_{t=1} = \frac{2.6829}{3.1072} = 0.863$$

An equation for the tangent line is $y - 4.621 = 0.863(x - 9.315)$

(c) The speed at $t = 1$ is $\sqrt{(x'(1))^2 + (y'(1))^2} = \sqrt{3.1072^2 + 2.6829^2} = 4.105$

(d) The acceleration vector is $\langle x''(1), y''(1) \rangle = \langle -28.425, 2.161 \rangle$