BC 5

$f'(x) = \frac{dy}{dx} = 1 - y$ and $f(1) = 0$ and $f(x) < 1$

(a) Euler's method with step size = -0.5

$$f(1) = 0$$
$$f(-0.5) \approx f(1) + f'(1)(-0.5) = 0 + (1-0)(-0.5) = -0.5$$
$$f(0) \approx f(-0.5) + f'(-0.5)(-0.5) = \boxed{-0.5 + (1-(-0.5))(-0.5)}$$
$$= -0.5 + (1.5)(-0.5) = -0.5 - .75 = \boxed{-1.25}$$

(b) Since $f(1) = 0$ and $1^3 - 1 = 0$, we can use L'Hopital's rule:

$$\lim_{x \to 1} \frac{f(x)}{x^3 - 1} = \lim_{x \to 1} \frac{f'(x)}{3x^2} = \frac{f'(1)}{3} = \frac{1-0}{3} = \boxed{\frac{1}{3}}$$

(c) $\frac{dy}{dx} = 1 - y$ and $f(1) = 0$

$$\frac{dy}{1-y} = dx$$

$-\ln|1-y| = x + C$ $\qquad$ $-\ln|1-0| = 1 + C \Rightarrow C = -1$

$$-\ln|1-y| = x - 1$$

$$\ln|1-y| = 1 - x$$

$|1-y| = e^{1-x}$ $\qquad$ Since, for this solution, $y = f(x) < 1$, then $|1-y| = 1-y$

$$1 - y = e^{1-x}$$

$$\boxed{y = 1 - e^{1-x}}$$