BC 5

$f(x)=\frac{1}{x} \quad g(x)=\frac{4x}{1+4x^2}$

(a) $g'(x)=\frac{4(1+4x^2)-4x\bullet 8x}{(1+4x^2)^2}=\frac{4-16x^2}{(1+4x^2)^2}=0$ for $x>0$ when $x=\frac{1}{2}$.

When $0<x<\frac{1}{2}$, $g'(x)>0$ and for $\frac{1}{2}<x<\infty$, $g'(x)<0$. So, $g(x)$ is increasing for $0<x<\frac{1}{2}$ and decreasing for $\frac{1}{2}<x<\infty$. Thus there is an absolute maximum on $(0,\infty)$ at $x=\frac{1}{2}$ and $\boxed{g\left(\frac{1}{2}\right)=1.}$

There is $\boxed{\text{no absolute minimum}}$ $(0,\infty)$ since $g(x)$ is increasing for $0<x<\frac{1}{2}$ and decreasing for $\frac{1}{2}<x<\infty$ and the endpoints of the intervals are not included.

(b) The area is given by the improper integral $\int_1^\infty\left(\frac{1}{x}-\frac{4}{1+4x^2}\right)dx=$

$$\lim_{b\to\infty}\left(\left(\ln x-\frac{1}{2}\ln(1+4x^2)\right)\Big|_1^b\right)=\lim_{b\to\infty}\left(\ln\left(\frac{x}{\sqrt{1+4x^2}}\right)\right)\Big|_1^b=$$

$$\lim_{b\to\infty}\left(\ln\left(\frac{b}{\sqrt{1+4b^2}}\right)-\ln\left(\frac{1}{\sqrt{5}}\right)\right)=\boxed{\ln\frac{1}{2}-\ln\left(\frac{1}{\sqrt{5}}\right)}$$