BC 6

$$f(x)=\begin{cases}\dfrac{\cos x-1}{x^2} & \text{for } x\neq 0\\ -\dfrac{1}{2} & \text{for } x=0\end{cases}$$

(a) $\boxed{\cos x = 1-\dfrac{x^2}{2!}+\dfrac{x^4}{4!}-\cdots \quad = \quad \sum_{n=0}^{\infty}\dfrac{(-1)^n x^{2n}}{(2n)!}}$

$$\cos x - 1 = -\frac{x^2}{2!}+\frac{x^4}{4!}-\frac{x^6}{6!}+\cdots$$

$$\boxed{f(x)=\frac{\cos x-1}{x^2}=-\frac{1}{2!}+\frac{x^2}{4!}-\frac{x^4}{6!}+\cdots \quad = \quad \sum_{n=0}^{\infty}\frac{(-1)^{n+1}x^{2n}}{(2n+2)!} \ \text{ or } \ \sum_{n=1}^{\infty}\frac{(-1)^n x^{2n-2}}{(2n)!}}$$

(b) $f'(x)=\dfrac{2x}{4!}-\dfrac{4x^3}{6!}+\ \cdots \ \Rightarrow \ f'(0)=0$

$$f''(x)=\frac{2}{4!}-\frac{12x^2}{6!}+\ \cdots \ \Rightarrow \ f''(0)=\frac{2}{4!}>0$$

$f(x)$ has a $\boxed{\text{relative minimum}}$ at $x=0$ because $f'(0)=0$ and $f''(0)>0$.

(c) $g(x)=1+\displaystyle\int_0^x f(t)\,dt \approx \boxed{1-\dfrac{x}{2!}+\dfrac{x^3}{3\cdot 4!}-\dfrac{x^5}{5\cdot 6!}}$

(d) $g(1)\approx \boxed{1-\dfrac{1}{2!}+\dfrac{1}{3\cdot 4!}}$

By the Alternating Series Estimation Theorem,

$$|\text{estimate} - \text{actual value}| < \frac{1^5}{5\cdot 6!}=\frac{1}{5\cdot 6!}<\frac{1}{6!}$$