Topological Lensing in Simply Connected Spherical Space

Abstract

Using no math beyond advanced high school calculus, this article seeks to show that an evolving and expanding, simply connected hyper-spheric universe embedded in 4D space will produce illusory images of sources as they evolve over time within the space. In the model, the view from the earth includes the galaxies, stars, and quasars within our current epoch and additionally contains an overlay of these same sources as they would have appeared billions of years ago. This overlay itself must also contain an overlay from an even earlier era, ad infinitum. Because of the long periods separating the ages of these images, sufficient change in the relative locations of their sources makes pairing or crystallographic methods of detection infeasible. A method for statistical analysis is sketched. The distribution of sources versus red shift provides a basis for detecting the implicit periodicity of the space, and hence its characteristic size.

Introduction

The idea that distant sources in the sky may be illusory is not new. Schwarzchild (1) raised the prospect even before he produced his early solutions to Einstein's field equations of general relativity. In addition, Friedman predicted the possibility of ghost images of astronomical sources in the context of his analysis of Einstein's theory. Friedman and Lemaitre are generally credited as discoverers of the concept of the big bang—conceiving it as an expanding hyper-sphere of three-dimensional space embedded in four dimensions based on Einstein's General Relativity. Einstein himself argued for a simply connected spherical topology. Recent work, (2) extends these ideas into the realm of ever more abstract and esoteric topology. This work is based on one key idea: topological lensing: if the geometry of the universe encompasses a real volume whose scale is smaller than the observed distance to some objects in a catalog of sources, then the catalog must contain multiple images of the some of the objects.

The curvature of the earth's surface is about 13 seconds of arc per mile. That is hardly enough to notice. Eratosthenes may be the first scientist to measure it (in 240 BC) and of course, Columbus applied the concept but radically misinterpreted his result. In a different way, the universe may be curved. Its curvature is perhaps 4 degrees of arc per billion light years. In this case, Einstein was the Eratosthenes and possibly Hubble is the Columbus. In any case, the implications of the curvature of the universe seem no more recognized today than were the implications of the earth's curvature recognized in Columbus' day.

There are many models of the geometry of the universe: the flat (Euclidean geometry) space used for example in explaining continuous creation, hyperbolic (non-Euclidean) and spherical (also non-Euclidean.) The curvature metric alone does not completely define the geometry of space. Thus, there are an enormous variety of topological variations: multiply connected models such as the simple torus and the so-called soccer ball model. (4)

This paper is not intended to argue against any of these but instead presents in some detail the observational consequences of what is arguably the simplest model of the geometry of a curvilinear universe, the completely symmetrical 3D hypersphere embedded in four dimensional space. While current literature and observational data seem to discount this model, there are two important reasons for considering it:

1) The math is quite simple--hence it might allow a more comfortable grasp of details which also correspond to those of other, more complex curvilinear cosmologies. Thus, it is hopefully the basis for a useful introduction to cosmological topology for a beginning student.

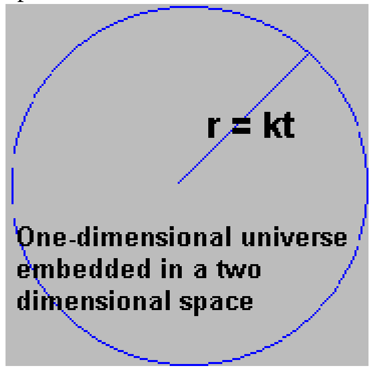
2) The conclusions seem profound. It is hard to believe that more sophisticated models do not result in some analog of these conclusions.

The conclusions are likely verifiable from observational data. An interesting speculation arises from the possibility that the conclusions are in verifiable in principle but in fact present an intractable combinatorial NP-complete challenge which cannot be solved with finite computing resources.

The model which is discussed in this paper is that of a 3-D hypersphere embedded in 4-D space. Time as an explicit dimension is left as an exercise for the reader. The 3D hypersphere-in-4space topology, of course, was the first solution to Einstein's field equations found by Willem de Sitter after Einstein introduced General Relativity(3.) Since de Sitter's model was obtained by making unrealistic assumptions about the universe (mass density = 0), it was generally taken not as a legitimate model of the real universe but as a toy model, only an example of the kind of solutions which could result from Einstein's fundamental theory. That point of view is also the spirit in which the reader is asked to read this paper.

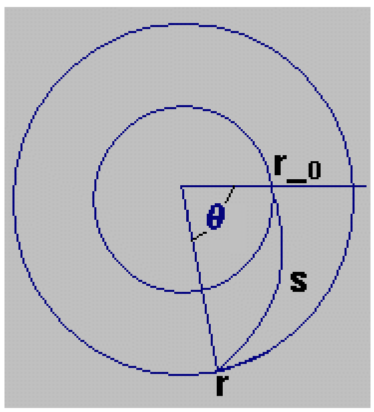
The central concept for understanding the three dimensional hypersphere embedded in 4-space is that of the line-of-sight. When we observe a star or other luminous source, it is seen as a point located by elevation and azimuth against the celestial globe, the object is located with these two dimensions. Curvature shows itself in the third dimension, the line-of-sight.

The path along the line-of-sight is the path that light follows. Geometrically, it is also the geodesic--the shortest distance between two points. In non-Euclidean space (curvilinear space) geodesics are, in some sense curved. Using the concept of line-of-sight to examine the mathematics of curved space--in this case closed expanding space--we are able to reduce the dimensionality of the situation to that of a one dimensional space: a circle (corresponding to the universe at any specific universal time,) embedded in a 2 dimensional space.



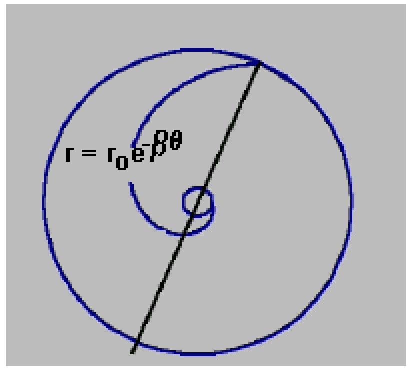
Thus, the 3-D hypersphere in 4-D space can be seen as a circle in a 2-D cross sectional slice though 4D hyperspace with the universe represented as a circle. The other two spatial dimensions map to elevation and azimuth. They are not needed for the analysis and can be re-introduced later.

A source (a galaxy or quasar) which is seen at an earlier time can be represented as located on a smaller circle concentric with a larger one representing the current universe. An observer will see the source along the line-of-sight from his position on the larger circle. This path is a geodesic in closed space. By symmetry, this space is only curved along the slice or cross-section we have chosen.



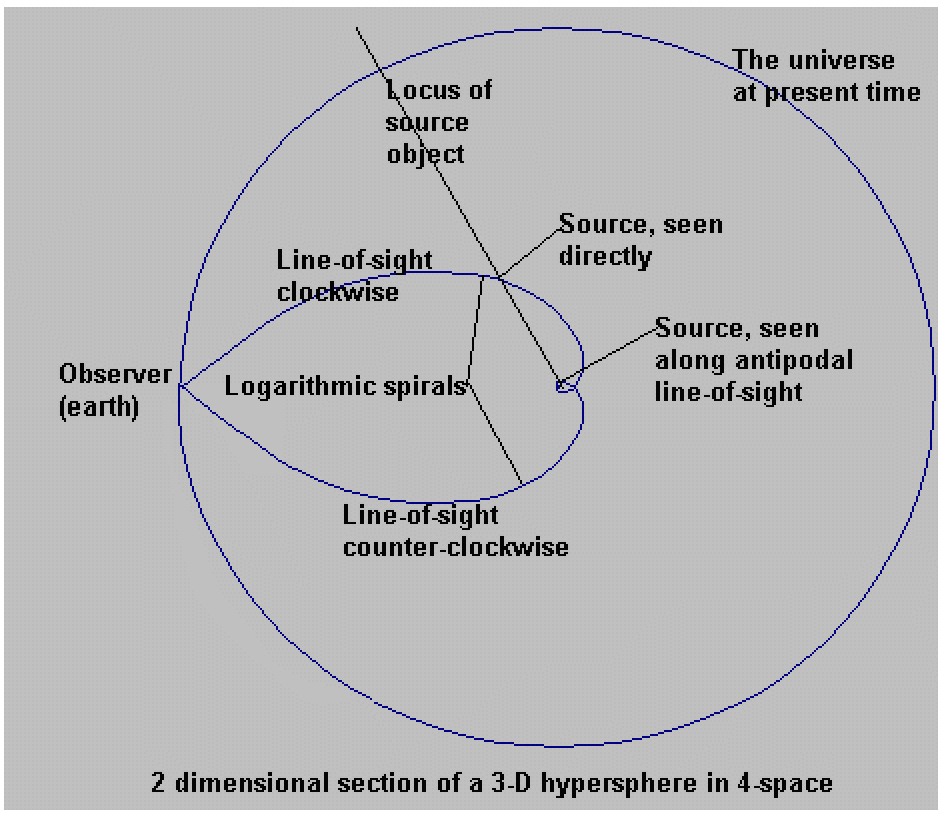
Using this view of the universe, an object located by the angle formed by the two lines--one between the observer and the center of the two circles, the other between the source and the center of the two circles--this angle is a measure of distance to the source from the observer. Call this angle theta. Note that the center of the two circles is not the center of the universe in the usual 3D sense. It corresponds to the location in the center of the 4D space from which all points in the embedded 3D hypersphere are equidistant.

Returning to angle we have just defined, theta is a convenient measure of the distance from the observer to the object. The distance in light-years is another measure however the angle theta, expressing distance in radians, an angular measure, provides an alternative way of locating an object.



It is relatively easy to show that the path followed by light, the geodesic connecting an

observer to a source, or the line of sight through the slice we have defined is a logarithmic spiral. (See Appendix A.) Since the space is closed, a distant source is seen both in the closest direction and some sense the opposite direction, light coming completely around the closed space and arriving at the observer. Classically this second view of the source has been called the antipode. There are two spirals connecting the observer to a source, one winding clockwise around the 4D hypersphere slice and the other clockwise. One connects the observer and source in the closest path, the other the furthest path.



These spirals are defined in polar coordinates as:

$r/r_b = e^{-beta*theta}$ where beta = sqrt((k/(c-k))^2 – 1)

where r is the radius of the universe at the source object —that is: at the time at which light just now being seen left the source. r_b is the radius of the universe at the time at which the light is now seen, (k/c) is the rate of expansion of the universe expressed as a fraction of the speed of light, and |theta| is the absolute value of the distance from the observer to the source expressed in the angular measure introduced in the paragraph above. In appendix B, we show that the rate of change of distance to the object when measured along the line-of-sight yields Hubble's constant. It is further useful to realize that the ratio: r/r_b expresses the fraction of the age of the universe at the time light from the source originated.

Returning to what might be regarded as the God's eye view of the universe over time: (the two dimensional slice though the 3d hypersphere embedded in 4space,) we can diagram the spiral line-of-sight as winding around the 4-space center. This spiral intersects a line drawn through the source and the 4-space center at multiple locations. Further, there are two spirals, clockwise and counter-clockwise. These locations account for where (or perhaps more precisely when) the source is when it is seen (detected) by the observer.

Along a single line-of-sight, any source will be seen at:

$$r/r_b = r_0 e^{-beta*theta} \quad theta > 0$$

and

$$r/r_b = r_0 e^{+beta*theta} \quad theta < 0$$

For thetas less than pi, the first equation represents the location of the source seen as closest to the observer while the second equation represents the apparent location of the source at its antipode: the object along the opposite line of sight.

As a matter of practicality, any source object has motion in addition to its radial movement due to expansion of the universe. Generally this motion may be small when compared with the overall expansion but with no loss in generality, recognize that while discussing the source as if it were at a constant specific value of theta, the next conclusion is equally valid if the source moved during the intervals at which its location coincides with the line of constant theta.

Looked at classically, the first image can be considered the source while the second image is a mirage, an image of the source from an earlier epoch.

Using this same reasoning, note that there are additional locations at which a value of theta will result in an image of the source along the line-of-sight.

In a closed universe, the primary image of a source object is at theta, r; where

$$r_o/r_b = exp(-beta*theta).$$

The antipodal image is located at a distance:

$$r_a/r_b = exp(-beta(pi - theta)$$

Combining these equations, we obtain the relation:

$$r_a*r_o = (r_b^2)\ e^{-beta\ pi}$$

This relates the age at which the antipode can be seen to the age at which the source object's primary image is seen. Note that conventionally, we would consider $r_o < r_a$, else the primary image should be the secondary. This leads to the recognition that:

$$r_a < r_b *sqrt(e^{(-beta* pi)})$$

Stated as a theorem:

No object in the sky can be further away than $r_b*B^{(beta/2)}$ where B = sqrt(exp(-pi); beta = sqrt((c/k)^2 – 1) and r_b is the age of the universe. All other imagery is an illusion produced by the optics of a 3D hypersphere embedded in 4-space. *or* **All the other imagery is light from the same objects from an earlier time.**

For example, if k/c = 0.2, then beta = 2 and assuming r_b = 15 billion years, the oldest real object are less than 0.2078796 * 15 billion or 3.12 billion years old.

If k/c = 1/sqrt(2), then beta = 1 and the oldest real objects are < sqrt(0.2078796)*15 = 6.8 billion years old.

There are alternative ways to explain the antipode effect discussed here. One is by analogy to the internal reflections of a spherical mirror. The other explains the effect as the images produced by gravitational lensing caused by the total mass of the entire universe. Both of these explanations rely on intuitive insight into the underlying principles of optics. Neither is useful if the audience lacks a similar perspective. This discussion grew out of an attempt to explain the phenomenon at the basic level using ray tracing. At that level, the need for analogy to mirrors or lenses disappeared and the direct approach of line-of-sight ray tracing is the result.

Use of the terminology: antipodes and mirages is the result of attempting to use optical metaphors to explain the effect. Using the line-of-sight ray-tracing model, it should be clear that a source object in a closed universe can be seen at multiple apparent locations. In the ray-tracing model, there is no special status for any of the various images that result. One, of course is viewed with the most recently transmitted light coming from the source. The additional images are the result of other paths. In the spherical mirror analogy, these would be called mirages.

Observational verification.

With such blatant conclusions, validating or negating the model with observational data should be straightforward. This actually appears not to be the case. There are many objects—even of galactic size and these objects will move enormous distances over periods of billions of years. Matching primary sources with their antipodal counter-parts is hardly easy. Conceptually, the process is relatively simple. For each object of a comprehensive catalog, calculate the value of theta for that object. Next, calculate the object's theoretical antipodal location and its value of theta. For any object and a candidate antipodal object, the angular separation of the two locations, expressed as sqrt((theta_1- theta_2)^2 + (azimuth_1-azimuth_2)^2 + (elevation_1 – elevation_2)^2) divided by (r_1 – r_2) is a measure of the velocity the object would need to have in order to move from where it was at the time of detecting its antipodal image to the position at which it is currently observed. Restated for clarity: the angular distance (which is expansion invariant) which an object moved to reach its observed location starting from the corresponding location when it was observed at its antipode when divided by the time available for that movement is an angular velocity which can be considered a measure of the likelihood that a candidate antipode can be associated with a given object. Using Bayesian hypothesis testing methods, the problem can be defined as one of finding which hypothesis is most likely to produce the results observed—in this case the hypotheses formed by 1) the set of mappings of objects to each other as antipode candidates and 2) values of beta. Preliminary formulations using a small sampling of objects/sources suggests that this approach will work. It seems to be close, but not entirely within the category of intractable combinatorial optimization problems such as the knapsack problem in OR. (reference: M. R. Garey and D. S. Johnson, "Computers and Intractability: A Guide to the Theory of NP Completeness," W. H. Freeman and Company, San Francisco, 1979.)

An alternative method would use the assumption that sources are uniformly distributed within the volume of the hypersphere. When viewed from a location on the hypersphere, the distribution of sources will appear to have a distribution which is a periodic function of the redshift. In this case, no pairing of sources is required.

John Bailey
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Appendix A
Equations for the line-of-sight
Logaritmic Spiral

The path of light from a source to the observer can be found by writing the differential condition it must satisfy, solving that expression for the equation of the path in polar coordinates, and setting the constants of integration to values which satisfy the boundary conditions.
Consider the path that light emitted from a shell at r_o must follow.

1) $(r\ dJ)^2 + (d\ r)^2 = (c\ dt)^2$

but k dt = dr

2) $(r\ dJ)^2 + (d\ r)^2 = (c\ dr/k)^2$

3) $(r\ dJ)^2 = (c\ dr/k)^2 - (d\ r)^2$

4) $r\ d\theta = \sqrt{((c/k)^2 - 1)}\ d\ r$

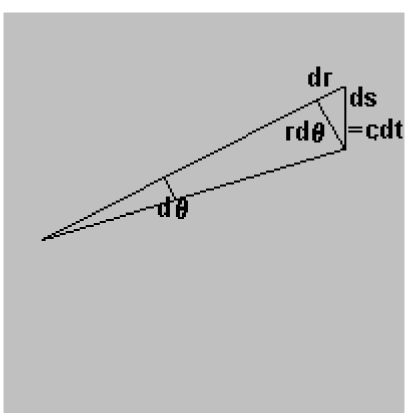
5) $d\theta = \sqrt{((c/k)^2 - 1)}\ d\ r/r$

integrating:

6) $\theta = \sqrt{((c/k)^2 - 1)}\ \log(r/r_o)$

7) let $beta = \sqrt{((c/k)^2 - 1)}$

8) $r/r_o = e^{-betaJ}$ The equation for a logarithmic spiral.



Appendix B
Deriving Hubble’s Constant

It may be helpful to show that Hubble’s constant can be derived from the relations developed thus far. We do this by calculating the distance from source to object along the line of sight with time not explicitly considered. We then use this expression for distance and consider how it must change as r_o changes. The result is the apparent recession of a distant source as a function of its distance from the observer.

Consider a differential length along the path of the line of sight.

1) $(r\ dJ)^2 + (d\ r)^2 = (ds)^2$

2) $beta^2 (dr)^2 + (dr)^2 = (ds)^2$ Substituting equation 4 from Appendix A

3) $ds = sqrt(beta^2 + 1)dr$ Collecting and re-ordering terms

4) $ds = c/k\ dr$ Using equation 7 appendix A, the definition of beta

integrating:

5) $s = c/k(r_o - r)$

6) $s = c/k\ r_o(1 - e^{-beta\ theta})$

Now consider how this length is changing over time.

7) $ds/dt = c/k\ (dr_o/dt)\ (1 - e^{-beta\ theta})$

8) $ds/dt = k(c/k)*(1 - e^{-beta\ theta})$

Let v represent the speed at which the end points are separating:

9) $v = c*(1 - e^{-beta\ theta})$

10) v = c*(1 - (r/r_0))

11) v = c*(r_0 - r)/r_0 = c(T_0 - T_s)/T_0

12) v = c*a / T_0

We have now obtained an expression involving
1) recession velocity v
2) speed of light, c
3) distance from the reference point (eg, the earth)
4) age of the universe, T_0

13) H = v/c*a = 1/T_0

v/c*a corresponds to Hubble's constant. T_0 is the age of the universe.

The reader is reminded, we are dealing here with a toy universe. Actual mileage may vary.
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